Adaptive Learning and Monetary Policy in Japan

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Abstract

This paper uses a dynamic stochastic open economy model to examine the welfare impact of monetary policy choices for Japan under both rational expectations and an adaptive learning framework. This setup allows us to assess systematically some of the debates concerning Japan's monetary policy actions in the past two decades and explore whether the public's expectation formation process may have contributed to the observed volatility of its economy. Focusing on a specific class of Taylor rules that react to observable data only, we find that: 1) an adaptive learning process may contribute to higher volatility in key economic variables; 2) a tight monetary policy rule that is overly sensitive to observed inflation creates excess volatility in general and incurs welfare cost; 3) explicit exchange rate stabilization is unwarranted from a welfare perspective; and 4) monetary policymaker may consider putting a stronger emphasis on domestic variables as its policy targets.

Keywords: Learning; Monetary Policy Rules; Open Economy

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1. Introduction

Japan's economic experience in the past two decades has attracted fervent research interests, be it on the liquidity traps, the optimal monetary and fiscal responses, or the structural dynamics of its underlying economy. On the empirical front, several papers point out that contrary to the experiences of other major OECD economies, Japan did not undergo a "great moderation" in the cyclical volatility of its real economic activity; rather, it may have switched from a moderate growth-low volatility regime to a low growth-high volatility regime.[1] Some attribute this empirical observation to policy mistakes. In particular, many argue that more desirable economic performance could have been achieved had BOJ's policy been less restrictive. Concerns have also been raised about the merits of BOJ engaging in exchange rate stabilization, rather than focusing solely on output and inflation targeting. With explicit micro-foundations, our general equilibrium model allows systematic evaluations of these arguments.

Besides potential policy mistakes, deviations from rational expectations may also contribute to economic volatility, as the public's expectation formation process can interact with either policy rules or structural shifts to influence the time path of the economy. A learning framework assumes that private agents are bounded rational. Rather than having full information, they only know the correct structure of the economy but have to rely on an adaptive learning process, such as least squares learning, to obtain information on the correct parameter values. As discussed in Williams (2003), since agents can only revise any expectation errors over time, this process may create high volatility and persistence in the economy. In addition, Orphanides and Williams (2004 and 2005) argue that a constant-gain learning framework can be employed to capture public agents' concern about potential structural change in the economy.[2] Given Japan's experiences in the past two decades – e.g. the bubble period and its subsequent burst – we see this as another motivation for incorporating learning dynamics into our analyses.

Our paper evaluates the welfare consequences of alternative monetary policy rules on the Japanese economy under rational expectations and also in a framework that allows

[1] See, for example, Stock and Watson (2005) and Yu (2005).
[2] Bullard and Duffy (2004) show that structural changes in the balanced growth path, interacting with adaptive learning with constant gain, contribute substantially to the observed variation in output in post-war US data.

adaptive agents to gradually learn the rational expectations equilibrium (REE).[3] Specifically, we apply the learning framework of Evans and Honkapohja (2001, hereafter EH) to a dynamic stochastic open economy model with nominal rigidity as in Gali and Monacelli (2005), hereafter GM (2005). We analyze the welfare performance of various specifications of the lagged-data-based Taylor rule, as done in the closed economy setting by EH (2003a) and Waters (2006).[4] Using as a benchmark a standard Taylor rule, with weights 1.5 and 0.5 on lagged inflation and output gap respectively, we compare the welfare outcomes of a tighter monetary policy rule that is more sensitive to inflationary pressure, and a rule that also specifically targets the terms of trade, reflecting exchange rate stabilization. Lastly, we consider a rule that targets *domestic producer price* inflation instead of *CPI* inflation. For each of these rules, we use a second order approximation of the representative consumer's utility function to compute the welfare losses under rational expectations, least squares learning and constant gain learning.[5]

Our simulation results show that first of all, a learning framework can lead to higher volatility in macroeconomic variables, but the result is sensitive to the specific policy rules. Second, regardless of the expectation formation process, a tight monetary policy rule relative to the standard benchmark leads to higher volatility in both output and *domestic producer price* inflation. Third, the policy rule that explicitly responds to the terms of trade creates substantially higher welfare cost than the rule that only reacts to the output gap and *CPI* inflation. This should not be surprising as *CPI* inflation already incorporates terms of trade movements, so additional exchange rate stabilization attempts would imply an overreaction by the policymakers, leading to excess volatility and welfare losses. These findings based on a structural general equilibrium model and

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[3] We restrict our analysis to the set of equilibria that is determinate and stable under learning, as our focus is on the quantitative importance of the learning, not on finding general conditions for learnable equilibria. Howitt (1992) and Bullard and Mitra (2002), among others, point out that the existence of a determinate REE should not be taken for granted as it is not clear whether or how economic agents can coordinate on that equilibrium. Monetary policy rules should thus pay attention to delivering a determinate REE which is learnable. Bullard and Mitra (2002) conclude that monetary policy rules obeying the "Taylor principle" could assure learnable equilibria. For a more detailed discussion on the conditions for determinacy and stability under learning for various classes of monetary policy, see Bullard and Mitra (2005), Evans and Honkapohja (2003a, 2003b, 2006) and Waters (2006). Llosa and Tuesta (2006) as well as Bullard and Schaling (2006) carry out similar analyses to an open economy setup.

[4] The set of Taylor rule that McCallum and Nelson (1999 and 2004) call "operational".

[5] The expected welfare losses of any policy rule that deviates from optimal policy can be approximated in terms of the variances of domestic inflation and the domestic output gap (see Woodford (2003) and Gali and Monacelli (2005), among many others).

systematic welfare evaluations in general support discussions in the literature that the high volatility of the Japanese economy may have resulted from too tight a monetary policy and/or BOJ's engagement in exchange rate stabilization. Finally, our last policy experiment shows that the *domestic producer price* inflation targeting rule dominates the *CPI* inflation targeting rule in terms of welfare ranking under all three rational expectations and learning equilibria. Thus it may be worthwhile to explore policy rules that place heavier emphases on stabilizing domestic variables only.

The paper is organized as follows. Section 2 reviews Japanese monetary policy during the past two decades. Section 3 outlines the model for the analysis of policy performances. Section 4 discusses the monetary policy rules. Section 5 explains the methodology, rational expectations and adaptive learning. Section 6 discusses calibration. Section 7 discusses our findings. Finally, section 8 concludes.

2. Japanese Monetary Policy

There has been a great deal of debate over the Bank of Japan's monetary policy during the bubble economy of the late 1980s and early 1990s as well as the economic downturn that followed. Several studies argue that more desirable economic performance should have been achieved had the Bank of Japan (BOJ) conducted less restrictive policy. Using a stylized simple Taylor rule with standard parameters as a benchmark, Bernanke and Gertler (1999), Jinushi, Kuroki and Miyao (2000) and McCallum (2000 and 2003) contend that BOJ's policy was too tight during much of the 1980s-1990s.[6] Some argue that the BOJ also slowly lowered the interest rate to accommodate the aftermath of the burst bubble began in 1992. In February 1999, the BOJ officially adopted the zero interest rate policy (ZIRP) under which the BOJ vowed to keep the call rate at zero until concern about deflation was dispelled.[7] The BOJ, however, temporarily abandoned the ZIRP by raising the call rate in March 2000 amid widespread criticism from many

[6] Standard parameters refer to the policy responses to any deviation of *CPI* inflation and the output gap from their target values are set to be 1.5 and 0.5, respectively. Also, the real interest rate is set to be 2 percent per annum. Generally, too tight monetary policy refers to when the actual instrument rate is above the target rate suggested by this standard Taylor rule.

[7] Jinushi, Kuroki and Miyao (2000) and Ito and Mishkin (2004) argue that the BOJ should have adopted the ZIRP earlier than February 1999.

economists and the government.[8] Ito and Mishkin (2004) call this interest rate hike as "a clear policy mistake."

One difficulty posed by using the Taylor rule to evaluate monetary policy is the various measurements of the output gap, which in turn leads to different policy implications.[9] To avoid this difficulty, McCallum (2003) adopts a monetary base rule to analyze Japanese monetary policy. Under this rule, policymakers set a change in monetary base to respond to any deviation of the growth rate of nominal GDP from its target and the average rate base velocity growth. Similarly, this rule suggests that BOJ policy was too tight all of the time since the mid 1990s.

Furthermore, some studies find that the BOJ also engages in exchange rate stabilization, rather than focusing solely on output and inflation targeting. Using the lagged specification of the Taylor rule responding to the real exchange rate movement, McKinnon and Ohno (1997) conclude that the BOJ systematically reacts to the yen/dollar real exchange rate during the period of October 1985-July 1995. They also document that the BOJ often adjusted the instrument rate to counter yen appreciation and promote yen depreciation. Similarly, Andrade and Divino (2005) conclude that the BOJ has implicitly targeted exchange rate stability, especially during the bubble period and its subsequent burst.[10] Yu (2005) finds that the BOJ stabilizes the yen/dollar real exchange rate using the short-term interest rate, which could explain high output volatility of the Japanese economy during the period 1993:Q1-2001:Q1.

Although a majority of the literature argues that the BOJ should have eased further after the bubble burst in the 1990s, the zero-lower bound on nominal interest rates created the difficulty that the short-term interest rate was no longer an effective policy instrument. Some researchers propose that the BOJ should expand the monetary base growth rate, rather than lower the call rate. However, when nominal interest rates are near zero, the purchase of short-term government bills using base money, through open market operations, will have no effect on asset market as short-term government bills and base money are almost perfect substitutes. An alternative rule suggested is to purchase unconventional assets such as long-term government bonds, foreign currencies or even

[8] The ZIRP was re-introduced in March 2001.
[9] See, for example, Ito and Mishkin (2004) and Kuttner and Posen (2004).
[10] See also Jinushi, Kuroki and Miyao (2000).

real estate. The BOJ followed this suggestion by raising the monthly purchase of long-term bonds from 400 billion yen to 1.2 trillion yen in several steps from August 2001 and October 2002. In addition, the purchase of foreign exchange which tends to depreciate the yen could lead to increases in net exports and then stimulates aggregate demand. Critics of this approach claim that this policy would hurt neighboring countries by reducing Japanese imports. McCallum (2003) counters that this policy would eventually increase net imports due to higher domestic income. McCallum thus proposes an exchange-rate targeting rule in which the yen/dollar real exchange rate should depreciate when inflation or output are below their target values.

3. The Model

In the present section we briefly outline and discuss the main equations of GM (2005) considered in our paper. The model is a small open economy model with Calvo-type staggered price-setting. Variables with a *H* subscript represent the *domestic* (*home*) economy and variables with a star superscript corresponds to the *world* economy.

The small open economy that is log-linearized about a steady state takes the following expressions

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\alpha}\left(r_t - E_t \pi_{H,t+1} - \overline{rr}_t\right) \quad (1),$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \kappa_\alpha x_t \quad (2),$$

where $\lambda \equiv \left[\left(1-\beta\theta\right)\left(1-\theta\right)/\theta\right]$, $\omega \equiv \sigma\gamma + \left(1-\alpha\right)\left(\sigma\eta - 1\right)$, $\sigma_\alpha \equiv \sigma / \left[1-\alpha+\alpha\omega\right]$ and $\kappa_\alpha \equiv \lambda(\sigma_\alpha + \varphi)$. x_t and r_t denote the output gap and the *domestic* interest rate, respectively. r_t can also be considered the policy instrument which is endogenous in the model. $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ is *domestic producer price* inflation where $p_{H,t}$ is the (log) domestic price index. The variable $\overline{rr}_t = \rho - \sigma_\alpha \Gamma(1-\rho_a)a_t + \alpha\sigma_\alpha(\Theta+\Psi)E_t\left[\Delta y^*_{t+1}\right]$ is the *domestic* natural rate of interest, where $\Gamma \equiv 1+\varphi/\left[\sigma_\alpha+\varphi\right]$, $\Psi \equiv -\Theta\sigma_\alpha/\left[\sigma_\alpha+\varphi\right]$ and $\Theta \equiv \omega - 1$. a_t is the labor productivity follows the AR(1) process of $a_t = \rho_a a_{t-1} + \varepsilon^a_t$.

Δy_{t+1}^* is the rate of growth of the level of *world* output. E_t represents private expectations that follow the mathematical expectation operator.

Equation (1) is a forward-looking version of the dynamic IS equation and equation (2) is a New-Keynesian Phillips curve (NKPC). Parameter β is the household discount factor, σ is the elasticity of intertemporal substitution (or the inverse of risk aversion), and φ is the inverse of labor supply elasticity. Parameter $\eta > 0$ measures the elasticity of substitution between domestic and foreign goods and γ measures the elasticity of substitution between imported goods. Parameter θ denotes a fraction of firms that keep prices unchanged which can also be interpreted as the degree of price stickiness, as suggested in Calvo (1983). Finally, the parameter $\alpha \in [0,1]$ represents the share of domestic consumption allocated to imported goods and can be interpreted as a degree of trade openness. It is interesting to note that when this small open economy is perfectly autarkic ($\alpha = 0$), the dynamic equations (1) and (2) are identical to the dynamic IS and NKPC equations, respectively, in a standard closed economy counterpart.[11]

Generally, the stylized Taylor rule assumes that policymakers react to *CPI* inflation. Thus we transform the structural equations (1) and (2) to explain the dynamic of *CPI* inflation, rather than *domestic producer price* inflation. Assuming that the *purchasing power parity* (PPP) condition holds, a relationship between *CPI* inflation (the rate of change in the CPI) and *domestic producer price* inflation (the rate of change in the index of domestically-produced goods prices) is given by

$$\pi_t = \pi_{H,t} + \alpha \Delta s_t \qquad (3),$$

where $s_t \equiv p_{F,t} - p_{H,t}$ is the (log) effective terms of trade, $p_{F,t}$ is the (log) price index for imported goods (expressed in domestic currency), $\pi_t = p_t - p_{t-1}$ is *CPI* inflation and p_t is the (log) consumer price index.[12]

[11] See, for example, Clarida, Gali and Gertler (1999) and Woodford (2003).

[12] Note that when the economy is completely autarkic, *CPI* inflation collapses to *domestic producer price* inflation. Thus the open economy model is identical to the closed economy counterpart.

Substituting (3) into (1) and (2), the structural equations characterizing the small open economy's equilibrium can be written as

$$x_t = E_t x_{t+1} - \frac{1}{\sigma_\alpha}\left(r_t - E_t \pi_{t+1} - \overline{rr}_t\right) - \frac{1}{\sigma_\alpha}\alpha E_t s_{t+1} + \frac{1}{\sigma_\alpha}\alpha s_t \qquad (4),$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa_\alpha x_t - \alpha\beta E_t s_{t+1} + \alpha\left(1+\beta\right) s_t - \alpha s_{t-1} + u_t \qquad (5).$$

As seen, the above two equations contain the terms of trade variable. We thus complement these two structural equations by introducing an additional equation describing the dynamic of the terms of trade.

Under the assumption of complete international financial markets, the log-linearized around a perfect foresight steady state version of *uncovered interest parity* (UIP) condition takes the form

$$r_t - r_t^* = E_t\left[\Delta e_{t+1}\right] \qquad (6)$$

where e_t is the (log) nominal effective exchange rate and r_t^* is the *world* interest rate. This equation implies that an expected depreciation (appreciation) of the nominal exchange rate must be offset by any positive (negative) interest rate differential between the *domestic* interest rate and the *world* interest rate – e.g. the opportunity cost of holding domestic currency rather than the other. Assuming that the *law of one price* holds for each individual goods, we obtain

$$p_{F,t} = e_t + p_t^* .$$

where p_t^* is the (log) *world* price index. Combining the previous expression with the definition of the (log) effective terms of trade, we obtain

$$s_t = e_t + p_t^* - p_{H,t} .$$

This expression also implies that

$$\Delta s_t = \Delta e_t + \pi_t^* - \pi_{H,t} \qquad (7)$$

where $\pi_t^* = p_t^* - p_{t-1}^*$ is *world* inflation. Combining (7) with the UIP condition (6), we obtain

$$s_t = E_t s_{t+1} - \left(r_t - E_t \pi_{H,t+1}\right) + \left(r_t^* - E_t \pi_{t+1}^*\right) \qquad (8).$$

Plugging (3) into (8), we obtain the following stochastic difference equation

$$(1-\alpha)s_t = (1-\alpha)E_t s_{t+1} - \left(r_t - E_t \pi_{t+1}\right) + \upsilon_t \qquad (9),$$

where $\upsilon_t = \left(r_t^* - E_t \pi_{t+1}^*\right)$ is a risk premium shock. Thus the small open economy is described by (4), (5), and (9), particularly when policymakers target *CPI* inflation.

We discuss now a losses function for analyzing the policy outcomes. Under the particular assumptions for which the strict domestic inflation targeting rule is optimal, GM (2005) derive a second order approximation to the domestic representative consumer's utility function.[13] This approximation represents the utility losses of any monetary policy that deviates from optimal policy and is expressed as a fraction of steady state consumption

$$\mathrm{W} = -\frac{(1-\alpha)}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{H,t}^2 + (1+\varphi)x_t^2\right] \qquad (10).$$

Taking unconditional expectations on (10) and letting $\beta \to 1$, the expected welfare losses of any policy rule that deviates from optimal policy rule can be expressed in terms of the variances of *domestic producer price* inflation and the output gap

$$EW = -\frac{(1-\alpha)}{2}\left[\frac{\varepsilon}{\lambda}\operatorname{var}(\pi_{H,t}) + (1+\varphi)\operatorname{var}(x_t)\right] \qquad (11).$$

We then use this approximation to evaluate the performances of (suboptimal) monetary policy rules by considering the resulting welfare losses.

4. Simple Monetary Policy Rules

This section discusses simple monetary policy rules for setting the *domestic* interest rate r_t.[14] We consider the lagged-data specification of a simple Taylor-type rule, an *operational* rule.[15] The Taylor rule can also be considered a suboptimal policy rule as r_t

[13] Optimal policy refers to policy that reproduces the flexible price equilibrium.

[14] Generally, the interest rate is also used to complement the dynamical system – e.g. for inflation and the output gap in the closed economy model.

[15] Evans and Honkapohja (2003b) also point out that the policy rule that responds to private expectations might not be an operational rule because policymakers may face a problem in availability of accurate observations on such expectations.

is set to respond to key macroeconomic variables without explicitly optimizing any policy objective function.[16]

First, we consider the *CPI* inflation Taylor rule (CITR). Under this policy rule, policymakers set the interest rate r_t to respond to *CPI* inflation and the output gap

$$r_t = \rho + \pi^T + \varphi_\pi(\pi_{t-1} - \pi^T) + \varphi_x x_{t-1} \tag{12}$$

where π^T is a target of *CPI* inflation. Parameters φ_π, φ_x > 0 measure how aggressive are the policymakers to any deviation of *CPI* inflation and the output gap from their target values, i.e. π^T and zero, respectively. Parameter $\rho = \beta^{-1} - 1$ is the time discount rate and could be interpreted as a quarterly riskless return in the steady state. It is important to note that reacting to *CPI* inflation also implies that the policymakers indirectly react to the terms of trade.

We consider next the policy rule that incorporate concerns about the Bank of Japan engages in exchange rate stabilization, besides focusing solely on output and inflation targeting. Under this policy rule, the policymakers also systematically respond to the terms of trade, reflecting exchange rate stabilization. This rule takes the form

$$r_t = \rho + \pi^T + \varphi_\pi(\pi_{t-1} - \pi^T) + \varphi_x x_{t-1} + \varphi_s s_{t-1} \tag{13}$$

where φ_s > 0 measures the response of the policymakers to the dynamics of the terms of trade.

The last policy rule considered is the *domestic producer price* inflation Taylor rule (DITR) under which the policymakers target *domestic producer price* inflation, rather than *CPI* inflation. This policy rule is given by

$$r_t = \rho + \pi_H^T + \varphi_{\pi_H}(\pi_{H,t-1} - \pi_H^T) + \varphi_x x_{t-1} \tag{14}$$

where π_H^T is a target of *domestic producer price* inflation and $\varphi_{\pi_H} > 0$ measures how aggressive are the policymakers to any deviation of *domestic producer price* from its target values.

[16] The Taylor rule is sometimes called an instrument rule. See Svensson (2003) as well as McCallum and Nelson (2004) for a survey on an instrument rule and a targeting rule.

5. Methodology

5.1 Rational Expectations

In this section we briefly describe a rational expectations framework under which private agents have perfect knowledge about the key aspects of the economy. We first complement the dynamical system given by (4), (5) and (9) with one of the monetary policy rules (12) or (13). Under the *domestic producer price* targeting rule (DITR), we complement the system given by (1) and (2) with a policy rule (14). Generally, the reduced form can be written as

$$y_t = \mathrm{A} + \mathrm{B}E_t y_{t+1} + \mathrm{C}y_{t-1} + \mathrm{D}w_t \qquad (15)$$

$$w_t = \rho_w w_{t-1} + \varepsilon_t \qquad (16)$$

where $y_t = \left[x_t , \quad \pi_t , \quad q_t , \right]'$, $w_t = \left[\overline{rr}_t , \quad u_t , \quad \upsilon_t , \right]'$, and $\varepsilon_t = \left[\varepsilon_{\overline{rr},t}, \quad \varepsilon_{u,t}, \quad \varepsilon_{\upsilon,t}, \right]'$ with appropriate matrices A, B, C, and D. The *Minimum State Variable* (MSV) solution to the system given by equations (15) and (16) takes the form

$$y_t = \overline{a} + \overline{b} y_{t-1} + \overline{c} w_t \qquad (17)$$

where $\overline{a}$, $\overline{b}$ and $\overline{c}$ are rational expectations equilibrium with conformable matrices.[17] In sum, under rational expectations with perfect knowledge, agents know the correct form of solution (17) and its relevant parameter values in matrices $\overline{a}$, $\overline{b}$ and $\overline{c}$.

5.2 Learning

This section discusses a learning methodology, proposed by EH (2001, 2003a, and 2003b). In contrast to rational expectations, the learning framework assumes that agents possess imperfect knowledge about the economy. Under learning private agents are bounded rational in the sense that they only know the correct structure of the economy but they rely on an adaptive learning process to obtain relevant parameter estimates. The fundamental idea of adaptive learning is that at each period *t* private agents possess the *Perceived Law of Motion* (PLM) whose form is analogous to the MSV solutions (17). Since the agents do not know the parameter values in matrices $\overline{a}$, $\overline{b}$ and $\overline{c}$, they estimate

[17] The MSV solution is generally considered a unique solution that is free of bubble and sunspot components. See McCallum (1983 and 1998).

their PLM – e.g. using least squares, to obtain parameter estimates of a_t, b_t, and c_t. Agents then perceive that the economy at time *t* would takes the form

$$y_t = a_t + b_t y_{t-1} + c_t w_t \qquad (18).$$

As in the learning literature, the exogenous shocks w_t are assumed to be observed by both agents and policymakers. By observing the current value of w_t, agents thus form their expectations using those parameter estimates and available information in hands up to and including period $t - 1$. This implies that

$$E_t y_{t+1} = a_t + b_t E_t y_t + c_t E_t w_{t+1}, \qquad \text{or}$$

$$E_t y_{t+1} = \left(I + b_t\right) a_t + b_t^2 y_{t-1} + \left(b_t c_t + c_t \rho_w\right) w_t . \qquad (19)$$

where ρ_w is also assumed to be known by agents. At each period *t*, the policymakers also set the interest rate r_t by following their desired rules. As a result, the *Actual Law of Motion* (ALM) for y_t is generated according to (15) and (16) and takes the following form

$$y_t = \mathrm{A} + \mathrm{B}\left[\left(I + b_t\right) a_t + b_t^2 y_{t-1} + \left(b_t c_t + c_t \rho_w\right) w_t\right] + \mathrm{C}\mathrm{Y}_{t-1} + \mathrm{D} w_t \text{ , or}$$

$$y_t = \left[\mathrm{A} + \mathrm{B}\left(I + b_t\right) a_t\right] + \left(\mathrm{B} b_t^2 + \mathrm{C}\right) y_{t-1} + \left[\mathrm{B}\left(b_t c_t + c_t \rho_w\right) + \mathrm{D}\right] w_t \qquad (20).$$ [18]

Accordingly, at the beginning of $t + 1$ agents use new available information – e.g. the previous data of relevant variables up to and including period *t*, to re-estimate the PLM and then obtain the parameter estimates a_{t+1}, b_{t+1} and c_{t+1}. Once the shocks w_{t+1} are realized and the interest rate r_{t+1} is set, by the policymakers, the ALM for y_{t+1} is generated and the learning process continues.

Under adaptive learning, the recursive least squares algorithm is given by

$$\phi_t = \phi_{t-1} + g_t R_t^{-1} z_{t-1} \left(y_{t-1} - \phi'_{t-1} z_{t-1}\right)' \qquad (21)$$

$$R_t = R_{t-1} + g_t \left(z_{t-1} z'_{t-1} - R_{t-1}\right)' \qquad (22)$$

where $\phi_t = \left[a_t, \quad b_t, \quad c_t\right]'$ and $z_t = \left[1, \quad y_{t-1}, \quad w_t\right]'$. R_t is the updated matrix of second moments of the regressors z_t. In sum, under adaptive learning the dynamics of the model

[18] Evans and McGough (2005) call the ALM as the true data generating process. Also, the ALM is sometimes called the temporary equilibrium for endogenous variables.

are defined by the recursive least squares updating equations (21) and (22), the expectations formation (19) derived from the PLM, the structural model equation (15). and the AR(1) process of stochastic shocks w_t (16).

The gain parameter g_t plays an important role in characterizing two types of adaptive learning. First, if the gain parameter is decreasing over time, $g_t = 1/t$, then the updating equations (21) and (22) are equivalent to recursive least squares using all lags. This type of learning is called least squares learning. Second, Orphanides and Willaims (2004 and 2005) argue that perpetual learning is more desirable in studying monetary policy performances. They model perpetual learning by replacing the decreasing gain parameter by a small constant gain, $0 < g_t < 1$ and call constant gain learning.[19] Under constant gain learning, agents make a learning process persistent with finite memory by placing more weight on recent data. Importantly, constant learning allows agents to remain alert to any potential structural change in the economy.[20] We then make use of these two types of learning for our analyses.

6. Calibration

In our calibration we partially adopt parameters proposed by GM (2005). That is we set the coefficient of risk aversion parameter σ, the elasticity substitution between imported goods γ and the elasticity substitution between foreign and domestic goods η to be 1. The probability of not adjusting prices θ and the inverse of the elasticity of labor supply φ are set to be 0.75 and 3, respectively. Parameter β is set to be 1, so $\rho = 0$.[21] The degree of openness parameter α is set to be 0.11 corresponding to the share of import to GDP in Japan during the period 1983:Q1-2005:Q2.

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[19] In their work, the gain is set in the range of zero and 0.1.

[20] Constant gain learning also implies that agents use rolling window regressions to update their expectations. A constant gain of g_t is equivalent to using $2/g_t$ lags of the data.

[21] This value of β is suggested in Ball (1999) and Mankiw and Reis (2002). Nunes (2004) argues that setting the discount factor to be one makes a zero output gap consistent with positive inflation at steady state.

For our benchmark policy, we set φ_π = 1.5 and φ_x = 0.5 suggested in Taylor (1993).[22] The target of *CPI* inflation, π^T = 0.822, is computed from the average of *CPI* inflation in Japan during the period 1983:Q1 – 2005:Q2. We also set φ_π = 2. This implies that the policy rule is more aggressive to deviations of *CPI* inflation form its target, which could be interpreted as too tight policy. For the policy rule responding to the terms of trade, we set φ_s = 0.2. The parameters φ_{π_H} and π_H^T in the DITR rule are set to be 1.5 and 0.822, respectively. In sum, we consider four monetary policy rules

Rule 1: $r_t = 0.822 + 1.5(\pi_{t-1} - 0.822) + 0.5x_{t-1}$ (CITR1)

Rule 2: $r_t = 0.822 + 2(\pi_{t-1} - 0.822) + 0.5x_{t-1}$ (CITR2)

Rule 3: $r_t = 0.822 + 1.5(\pi_{t-1} - 0.822) + 0.5x_{t-1} + 0.2s_{t-1}$ (CITR3)

Rule 4: $r_t = 0.822 + 1.5(\pi_{H,t-1} - 0.822) + 0.5x_{t-1}$ (DITR).

We next discuss the properties of the stochastic shocks. First, we generate the process for $\overline{rr}_t$ using its definition shown in section 3.[23] Fitting the AR(1) process to $\overline{rr}_t$ gives

$$\overline{rr}_t = 0.66\overline{rr}_{t-1} + \varepsilon_t^{\overline{rr}}, \quad \text{with standard deviation of } \varepsilon_t^{\overline{rr}} = 0.0029.$$

The foreign exchange risk-premium υ_t is computed from fitting the AR(1) process to the US real interest rate over the same period.[24] The stochastic process of υ_t takes the following form

$$\upsilon_t = 0.97\upsilon_{t-1} + \varepsilon_t^{\upsilon}, \quad \text{with standard deviation of } \varepsilon_t^{\upsilon} = 0.005.$$

Finally, we specify the stochastic process of the domestic cost push shocks u_t. We consider four processes which are *i.i.d.* or AR (1) processes with low or high volatility.[25] Specifically, u_t takes one of the following forms

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[22] Recall that this benchmark policy rule is often used for the analysis of whether BOJ policy is too tight.

[23] With our parameters considered, $\overline{rr}_t = \rho - \sigma_\alpha \Gamma(1-\rho_a)a_t$. a_t is labor productivity in log deviations from a linear trend. We use the Japanese labor productivity during the same period obtained form Source OECD.

[24] We follow the methodology proposed by Monacelli (2004).

[25] The *i.i.d.* process of cost push shocks is suggested by Svensson (2000).

1) $u_t = \varepsilon_{u,t}$, with standard deviation of ε_t^u = 0.001,

2) $u_t = \varepsilon_{u,t}$, with standard deviation of ε_t^u = 0.01,

3) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$, with standard deviation of ε_t^u = 0.001, or

4) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with standard deviation of ε_t^u = 0.01.

7. Simulation Results

To compare the welfare impact of alternative monetary policy rules, we conduct our simulation exercise following the learning algorithm described in EH (2001 and 2006). In order to ensure comparability of welfare outcomes across policy rules, we impose the same the disturbances across simulations, as suggested by Carceles-Poveda and Brook (2006).[26] The initial conditions for each simulation are set to be the rational expectations equilibrium values, augmented with an additional noise to: $a = \bar{a} + 0.005 \times \text{random}$, $b = \bar{b} + 0.04 \times \text{random}$, $c = \bar{c} + 0.02 \times \text{random}$, $R = \bar{R}$, and $y_0 = \bar{y}$, where random is normally distributed disturbance term.

In the least squares algorithm, we mitigate the initial volatility of parameters estimates by using a small constant gain for the first 20 periods. That is, $g_t = 1/N$ for t = 1, 2, …, N and $g_t = 1/t$ for $t > N$, with N = 20. We then simulate the dynamics of the economy for 200 periods and evaluate the performance of policy rules based on welfare losses measured as the sum of the variances of the output gap and *domestic producer price* inflation.

Table 1 reports the welfare losses of the policy rules CITR1 and CITR2 under rational expectations, least squares learning and constant gain learning. Recall that the rule CITR1 is benchmark policy. The second and third columns show the performances of policy rules under rational expectations and least squares learning, respectively. The fourth through sixth columns reports the policy outcomes under constant gain learning for small gain values, g_t, of 0.01, 0.02, and 0.05. All numbers are percentage of steady-state

[26] We run simulations using state values 56, 46 and 64 to generate normally distributed disturbance terms in Matlab. Our results are robust to these all these scenarios considered.

consumption and in deviation from optimal policy. Numbers in parentheses represent the percentage of the output gap variation contributing to the losses.

As shown in Table 1, for all simulations the welfare losses under rational expectations are substantially different from those under learning. In particular, under adaptive learning the volatility of *domestic producer price* inflation and the output gap is higher than that of under rational expectations. This result is contrary to Williams (2003) finding that with a simple Taylor rule in a simple New-Keynesian closed-economy model the volatility of the output gap and inflation under adaptive learning is similar to that of under rational expectations. Our finding implies that adaptive learning may induce higher volatility in some economic variables.

One outstanding result is that regardless of the expectations formation process the rule CITR2, a tight policy rule relative to the benchmark policy rule CITR1, leads to undesirable fluctuations in both output and *domestic producer price* inflation. This in turn contributes to higher welfare losses under all four types of cost push shocks analyzed. Furthermore, the rule CITR2 performs very poorly when agents are learning. For example, under least squares learning with high volatility of cost push shocks (the second panel) the loss of the policy rule CITR2 is approximately 66.34 percent higher than that of the rule CITR1 while under rational expectations with the same cost push shocks the loss of the rule CITR2 is just 36.22 percent higher than that of the rule CITR1. In addition, under all scenarios the welfare losses of both rules are substantially attributed to the variation in the output gap. Particularly, under the policy rule CITR2, more than 90 percent of the losses can be explained by output gap variation.

Table 2 reports the welfare losses associated with the rule CITR3, under which policymakers explicitly respond to the terms of trade. The layout of Table 2 is similar to that of Table 1. For comparison, the welfare losses of the rule CITR 1 are reported in the corresponding model. As shown, regardless of how agents form their expectations the rule CITR3, reflecting exchange rate stabilization, performs very poorly as it substantially amplifies the welfare costs relative to the benchmark policy rule CITR1 in which policymakers reacts only to the output gap and *CPI* inflation. This finding should not be surprising because *CPI* inflation already incorporates terms of trade movements, $\pi_t = \pi_{H,t} + \alpha \Delta s_t$, so additional attempts to stabilize the exchange rate may imply an overreaction by the policymakers, leading to excess volatility in output and then higher welfare losses. Similarly, the volatility of the output gap, approximately 92 percent, mainly attributes to the welfare losses. Since the policy rules CITR2 and CITR3 create quantitatively higher welfare losses, mainly resulting from substantially high output variation, we conclude that with a structural general equilibrium model and systematic welfare evaluations the high volatility of the Japanese economy may have resulted from too tight monetary policy and/or BOJ's engagement in exchange rate stabilization.

Table 3 reports the welfare losses associated with the policy rule DITR that the policymakers respond to the output gap and *domestic producer price* inflation, instead of *CPI* inflation. The format of Table 3 is also similar to that of Table 1. For comparison, we report the outcomes of the policy rule CITR1. As shown, for all simulation the welfare losses of the rule DITR can be mainly explained by the volatility of the output gap. Under all scenarios the rule DITR significantly outperforms the rule CITR1, and then policy rules CITR2 and CITR3. In particular, with low volatility of cost push shocks (the first and third panels) the losses of the rule CITR2 are much smaller than those of the rule CITR1. For example, with low volatility of AR(1) cost push shocks (the third panel), the losses of the rule DITR are 94.60 percent and 97.52 percent below those of the rule CITR1 under rational expectations and least squares learning, respectively. In sum, the *domestic producer price* inflation targeting rule dominates the *CPI* targeting rule in terms of welfare ranking under both rational expectations and adaptive learning. Thus it may worthwhile for the policymakers to consider putting stronger emphases on domestic variables as their policy targets.

8. Conclusion

With explicit micro-foundations, our general equilibrium model allows systematic evaluations of arguments about Japanese monetary policy during the past two decades – e.g. too tight policy and exchange rate stabilization policy. Since an adaptive learning framework allows agents to update their expectation errors over time, it may create high volatility and persistence in the economy. We thus see this as a motivation for incorporating learning dynamics into our analyses of high output variation of the

Japanese economy in the past two decades. We evaluate the welfare consequences of various types of the Taylor rules under rational expectations and adaptive learning framework. Specifically, we make use of a second order approximation of the representative consumer's utility function to compute the contribution to the welfare losses.

Our first finding is that a learning framework can create higher volatility in macroeconomic variables, but the result is sensitive to the specific policy rules. Second, a tight policy rule relative to the standard Taylor rule, our benchmark policy rule, leads to undesirable fluctuations in both output and *domestic producer price* inflation. Next, the policy rule systematically responding to the terms of trade, implying exchange rate stabilization, substantially amplifies the welfare costs relative to the benchmark policy rule. These two findings in general support discussions in the literature that the high volatility of the Japanese economy may have resulted from too tight a monetary policy and/or BOJ's engagement in exchange rate stabilization. Finally, our last policy experiment shows that the *domestic producer price* inflation targeting rule outperforms the *CPI* inflation targeting rule in terms of welfare ranking. Thus it may be worthwhile to explore policy rules that put a stronger emphasis on stabilizing domestic variables.

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Table 1: Contribution to Welfare Losses of Policy Rules CITR1 and CITR2

Policy Rule	Rational Expectations	Least Squares Learning	Constant Gain Leaning		
			g_t = 0.01	g_t = 0.02	g_t = 0.05
1) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0248	0.0858	0.0703	0.0681	0.0549
	(83.47)	(77.51)	(69.70)	(72.10)	(77.23)
CITR2	0.0354	0.1161	0.1227	0.1162	0.0691
	(86.44)	(91.30)	(88.75)	(89.76)	(89.87)
2) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0370	0.1025	0.0831	0.0803	0.0633
	(77.57)	(79.12)	(72.32)	(73.10)	(76.46)
CITR2	0.0504	0.1705	0.1427	0.1309	0.0906
	(83.73)	(89.21)	(88.79)	(88.77)	(86.75)
3) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0250	0.0766	0.0679	0.0646	0.0459
	(83.60)	(81.59)	(70.69)	(73.84)	(79.96)
CITR2	0.0357	0.1359	0.1254	0.1083	0.0732
	(86.55)	(92.49)	(89.71)	(90.86)	(90.85)
4) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0502	0.1011	0.0960	0.0913	0.0714
	(749.0)	(77.15)	(72.50)	(73.16)	(75.35)
CITR2	0.0649	0.1572	0.1539	0.1366	0.0961
	(83.51)	(88.61)	(88.30)	(88.07)	(86.68)

Note: Numbers are percentage of steady-state of consumption. Numbers in parentheses represent percentage of the volatility of the output gap that contributes to the welfare losses.

Table 2: Contribution to Welfare Losses of Policy Rules CITR1 and CITR3

Policy Rule	Rational Expectations	Least Squares Learning	Constant Gain Leaning		
			g_t = 0.01	g_t = 0.02	g_t = 0.05
1) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0248	0.0858	0.0703	0.0681	0.0549
	(83.47)	(77.51)	(69.70)	(72.10)	(77.23)
CITR3	3.1293	2.8925	2.8302	2.8438	2.9146
	(92.81)	(91.59)	(92.72)	(92.24)	(92.16)
2) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0370	0.1025	0.0831	0.0803	0.0633
	(77.57)	(79.12)	(72.32)	(73.10)	(76.46)
CITR3	3.1381	2.8846	2.8605	2.8667	2.8987
	(92.80)	(91.74)	(91.82)	(91.93)	(92.24)
3) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0250	0.0766	0.0679	0.0646	0.0459
	(83.60)	(81.59)	(70.69)	(73.84)	(79.96)
CITR3	3.1294	2.8993	2.8341	2.8424	2.9130
	(92.80)	(92.21)	(92.32)	(92.47)	(92.61)
4) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0502	0.1011	0.0960	0.0913	0.0714
	(74.90)	(77.15)	(72.50)	(73.16)	(75.35)
CITR3	3.1504	2.9032	2.8774	2.8815	2.9217
	(92.46)	(91.13)	(91.55)	(91.71)	(91.82)

Note: Numbers are percentage of steady-state of consumption. Numbers in parentheses represent percentage of the volatility of the output gap that contributes to the welfare losses.

Table 3: Contribution to Welfare Losses of Policy Rules CITR1 and DITR

Policy Rule	Rational Expectations	Least Squares Learning	Constant Gain Leaning		
			g_t = 0.01	g_t = 0.02	g_t = 0.05
1) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0248	0.0858	0.0703	0.0681	0.0549
	(83.47)	(77.51)	(69.70)	(72.10)	(77.23)
DITR	0.0014	0.0018	0.0019	0.0018	0.0016
	(78.57)	(66.67)	(73.68)	(72.22)	(75.00)
2) $u_t = \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0370	0.1025	0.0831	0.0803	0.0633
	(77.57)	(79.12)	(72.32)	(73.10)	(76.46)
DITR	0.0122	0.0126	0.0128	0.0126	0.0123
	(54.92)	(54.76)	(55.47)	(55.56)	(54.47)
3) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.001					
CITR1	0.0250	0.0766	0.0679	0.0646	0.0459
	(83.60)	(81.59)	(70.69)	(73.84)	(79.96)
DITR	0.0016	0.0019	0.0021	0.0019	0.0017
	(68.75)	(68.42)	(66.67)	(68.42)	(70.59)
4) $u_t = 0.4u_{t-1} + \varepsilon_{u,t}$ with SD of $\varepsilon_{u,t}$ is 0.010					
CITR1	0.0502	0.1011	0.0960	0.0913	0.0714
	(74.90)	(77.15)	(72.50)	(73.16)	(75.35)
DITR	0.0289	0.0298	0.0300	0.0296	0.0290
	(57.44)	(57.72)	(57.67)	(57.77)	(57.59)

Note: Numbers are percentage of steady-state of consumption. Numbers in parentheses represent percentage of the volatility of the output gap that contributes to the welfare losses.