Forecasting Inflation using the Term Structure and MARS

Abstract

This note revisits the issue of forecasting changes in inflation using non-linear non-parametric methods. The results indicate the presence of threshold effects in the relationship between the information in the term structure and changes in the rate of inflation.

JEL C14, C59, E47

Keywords Inflation, term structure, non-parametric estimation

1. INTRODUCTION

Central bankers across North America have adopted targets for short-term interest rates as their mechanism for implementing market-conditioning monetary policy. Recent history indicates that there is substantial information conveyed to markets through changes in targets for the US federal funds rate, as well as the target band for the overnight financing rate set by the Bank of Canada. Whether these shifts in the target rate or the target band are in response to market conditions, or the final result of "watchful waiting" as might have been suggested by Poole (2001), they are ultimately aimed at sheltering the economy from the vagaries of the business cycle.

One might wonder whether recent interest rate reductions in the face of eroding business confidence were inconsistent with the wisdom of setting monetary conditions to provide a low and stable rate of inflation in the long run. This is particularly true given that the inflation rate appeared to be near the upper end of the "acceptable range" for most central bankers in the early part of this century. Indeed, during the unprecedented expansion of the 1990s conventional reasoning suggested that increases in interest rates were required to maintain inflation within its target range. Now that interest rates have fallen to historic lows and appear "wobbly", one might wonder if demand pressures will re-ignite and push inflation further into the danger zone.

In a series of papers, Mishkin (1990a, 1990b, 1991) employed Fisher equations to demonstrate that the slope of the term structure could explain changes in the inflation rate. During the expansion of the 1990s, many economists believed that an increase in the slope of the term structure would restrain inflation: it was possible to maintain a low and stable inflation rate, thereby generating the associated benefits.

Tkacz (2000) revisited this issue to determine whether there were non-linearities in the relationship between the slope of the term structure and changes in the inflation rate. This was justified by appealing to evidence suggesting asymmetric effects of monetary policy either within the context of a non-linear Phillips curve, or more generally within a non-linear model of investment and banking behaviour. Using neural network models, Tkacz (2000) demonstrated that a substantial tightening of the term structure was required to maintain changes in the inflation rate at low and stable levels for all policy horizons.

The purpose of this paper is to revisit the data to determine whether an alternative approach to modelling provides a similar result. The findings suggest that there are threshold effects in the link between changes in inflation and interest rate spreads and that a reduction in the spread does not uniformly reduce the inflation rate over time. This suggests that monetary policies aimed at maintaining low and stable inflation need to be set with a view to the presence and location of these threshold effects.

2. CHANGES IN INFLATION AND THE TERM STRUCTURE ON MARS

The problem facing any modeller is to determine the fundamental relationship between a dependent variable and a vector of predictors, expressed by X.[1] Specifically, the question is how best to capture the functional form f{.} in equation (1):

$$Y = f\{ X \} + \varepsilon \qquad (1)$$

where ε captures the departure of the dependent variable (Y) from the equilibrium relationship linking X to Y. In the present context, Y denotes the change in the inflation rate at time t ($B_t^M - B_t^N$), while X represents the difference between the M-period nominal interest rate and the N-period nominal interest rate ($I_t^M - I_t^N$) at time t.[2]

The idea behind local non-parametric modeling is to allow for a potentially non-linear relationship over different ranges of X. Friedman (1991a, 1991b) introduced the multivariate adaptive regression splines (MARS) approach of using smoothing splines to fit the relationship between a set of predictors and a dependent variable. By requiring the curve segments to be continuous (so that first and second derivatives are non-zero), one obtains a very smooth line that can capture "shifts" in the relationship between variables. These shifts occur at locations designated as "knots", and provide for a smooth transition between "regimes". The MARS algorithm searches over all possible knot locations, across all variables and all interactions among all variables. It does so through the use of

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1 This section draws heavily on work presented by Sephton (2001).

2 The M-period inflation rate is defined as log (P_{t+1}/P_t)*(1200/M) where M takes on values 3,6,9,12,36,60, and 120. The yield spreads and inflation changes always involve the difference between the long-run and the short-run. Hence ($I_t^{120} - I_t^3$) denotes the difference between the yields on 10 year government bonds and three month treasury bills.

combinations of variables called “basis functions”, which are similar to variable combinations created by using principal components analysis. Once MARS determines the optimal number of basis functions and knot locations, a final least squares regression provides estimates of the fitted model on the selected basis functions.

When modeling the relationship between a single predictor X_t and the dependent variable Y_t, a general model might take the form

$$Y_t = \sum_{k=1}^{M} \alpha_k B_k(X_t) + \varepsilon_t \qquad (2)$$

where $B_k(X_t)$ is the k^{th} basis function of X_t. Basis functions can be highly non-linear transformations of X_t, but note that Y_t is a linear (in the parameters) function of the basis functions. Estimates of the parameters α_k are chosen by minimizing the sum of squared residuals from equation (2). The advantage of MARS is in its ability to estimate the basis functions so that both the additive and the interactive effects of the predictors are allowed to determine the response variable.

MARS identifies the knot locations that most reduce the sum of squared residuals. For example, with a single predictor the sum of squared residuals would be

$$\sum_{i=1}^{N} \{ \; Y_t \; - \; \sum_{j=0}^{Q} b_j \, X_t^{\,j} \; - \; \sum_{k=1}^{K} a_k \; (X_t - t_k)_+^{Q} \; \}^2 \qquad (3)$$

where b_j and a_k are multiple regression coefficients on cubic (Q=3) splines of X_t , and X_t relative to knot location t_k. The notation $(X_t - t_k)_+^{Q}$ indicates that the cubic spline of X_t relative to knot location t_k is included if the difference is positive, otherwise it is zero.

From (3) it is clear that the addition of a knot can be viewed as adding the corresponding variable $(X_t - t_k)_+^{Q}$. A forward and backward stepwise search is incorporated in the MARS algorithm, with the forward step purposely over-fitting the data. Insignificant terms are deleted on the backward step of the routine.

Here, model selection is based on the generalized cross-validation (GCV) criterion of Craven and Wahba (1979), although in practice, any model selection criterion could be used. The GCV can be expressed as

$$GCV = (1/N) \sum_{t=1}^{N} \{ [Y_t - f_M (X_t)]^2 \; / \; [1 - C(M)/N]^2 \} \qquad (4)$$

where there are N observations, and the numerator measures the lack of fit on the M basis function model $f_M (X_t)$. This term corresponds to the sum of squared residuals from the fitted model. The denominator contains a penalty for model complexity, C(M), which is related to the number of parameters estimated in the model.

MARS estimates can most readily be interpreted from an ANOVA (analysis of variance) representation of the model, where the fitted function is expressed as a linear combination of additive basis functions in single variables and interactions between variables. MARS provides graphical plots which illustrate the optimal transformation of the variables chosen by the algorithm, much like the ACE (alternating conditional expectations) algorithm of Brieman and Friedman (1985).[3] As part of the MARS output, the relative contribution of each variable is determined, as are estimates of the model's adjusted R-squared given that a particular ANOVA function (variable) has been omitted from the model. This assists in interpreting the significance of each ANOVA function.[4]

MARS models were estimated for the change in the inflation rate over a number of policy horizons: three years relative to three months; three years relative to six months; three years relative to twelve month; five years relative to three months; five years

[3] The ACE approach to modeling finds the non-linear transformation of the predictors which maximizes the correlation between the dependent variable and the transformed predictors. A plot of the transformed series against the dependent variable is sometimes helpful in identifying a functional form to be used in parametric modeling. Hallman (1990) and Granger and Hallman (1991) employed ACE to examine non-linear cointegration.

[4] Since there is only one predictor in this model there is no question of which variables to allow to interact. Subsequent work incorporating additional factors into an inflation forecasting equation may demonstrate the sensitivity of the MARS algorithm to the degree of variable interaction allowed.

relative to six months; five years relative to twelve months; and finally ten years relative to the three-, six- and twelve-month horizons.

A large amount of output is produced when fitting each specification and it would be of little merit to present the results of each model. For the sake of brevity, only three of the estimated models will be discussed here (all figures and models may be obtained from the author on request). Of primary interest are the plots of the optimal transformation of the information in the yield curve as it relates to changes in the inflation rate. For example, Figure 1 presents the transformed spread between three year and three month interest rates and the associated changes in the inflation rate between three months and three years. The estimated thresholds in the spread are at 0.22 percent and –0.61 percent, respectively. As the spread rises above 0.22 percent, the inflation rate three years hence will rise linearly relative to the inflation rate in three months time. Alternatively, a tightening of the spread over this time horizon will reduce the inflation rate, but only until the spread reaches 0.22 percent. As the spread falls and is between –0.61 percent and 0.22 percent, there is a significant reduction in inflation over this policy horizon. Values of the spread below –0.61 percent (ie, an inverted yield curve) do not appear to be able to reduce further the three-year inflation rate.

Figure 2 presents the five year – three month estimates, with estimated thresholds at 0.22 percent and –0.82 percent. As in the three year - three month horizon, the relationship between the change in the inflation rate and the yield spread is non-linear. Spreads below the lower threshold have little effect on the change in the inflation rate. A substantial tightening is necessary to reduce inflation when the yield curve is upward sloping, but an inverted yield curve demonstrates that inflation can fall substantially if the short term rate changes marginally. Note that the upper threshold effect over the five year horizon is identical to that estimated for the three year time frame.

Finally, Figure 3 plots the optimal transform between the ten year and three month specifications. It is clear that there is little impact of a change in the slope of the term structure on the long-run inflation rate. Estimated thresholds are at 0.28 percent and –1.27 percent, with linear fits appearing to rotate at the knot locations. One might expect this result over such a long time frame given that there are a myriad of other factors contributing to the evolution of inflation, which are left unexplained by this approach to inflation forecasting.

3. CONCLUSIONS

Common to each figure is a non-linear transform which appears to indicate that a tightening of the yield spread will reduce the inflation rate over a three year to five year horizon relative to its value in three months. A substantial tightening may be required to reduce inflation when the yield curve is upward sloping. Inverted yield curves appear to provide significant reductions in inflation for small changes in the yield spread. Common estimates of an upper threshold for the spread of between 0.22 and 0.28 percent suggest there is a relatively robust non-linearity which needs to be considered when adjusting interest rate targets to achieve gains in inflation stability.

Subsequent work might incorporate other factors contributing to changes in inflation (commodity prices, changes in regulated prices) to determine the degree to which monetary policy is capable of reducing inflation to a low and stable long-run path. These findings suggest that non-linear non-parametric methods should be added to the applied econometrician's toolkit.

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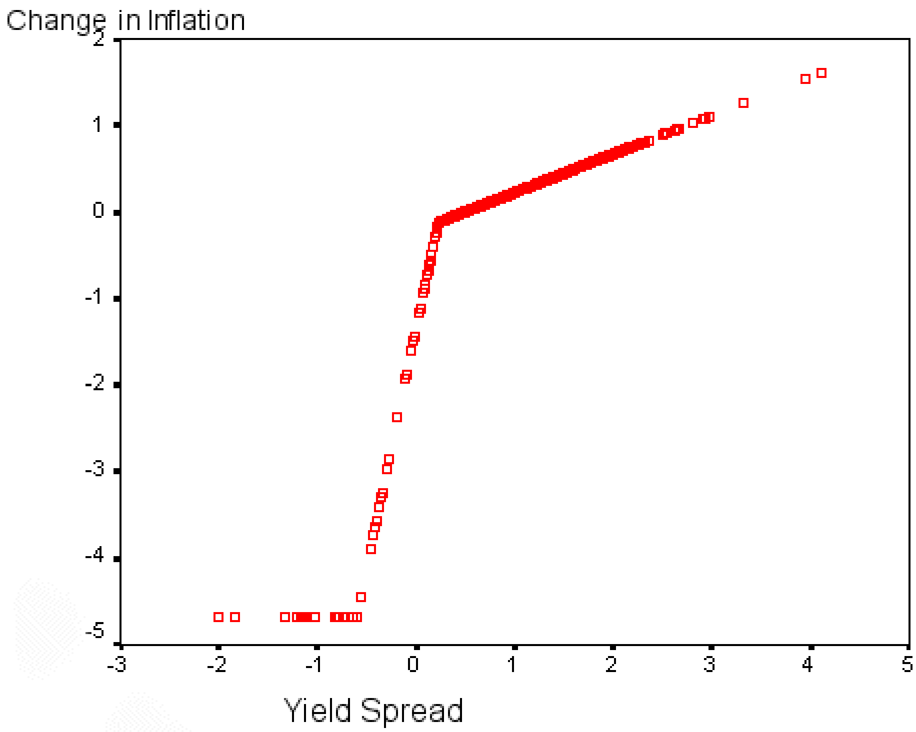

Figure 1: Yield Spread and Inflation Changes: Three Years vs Three Months

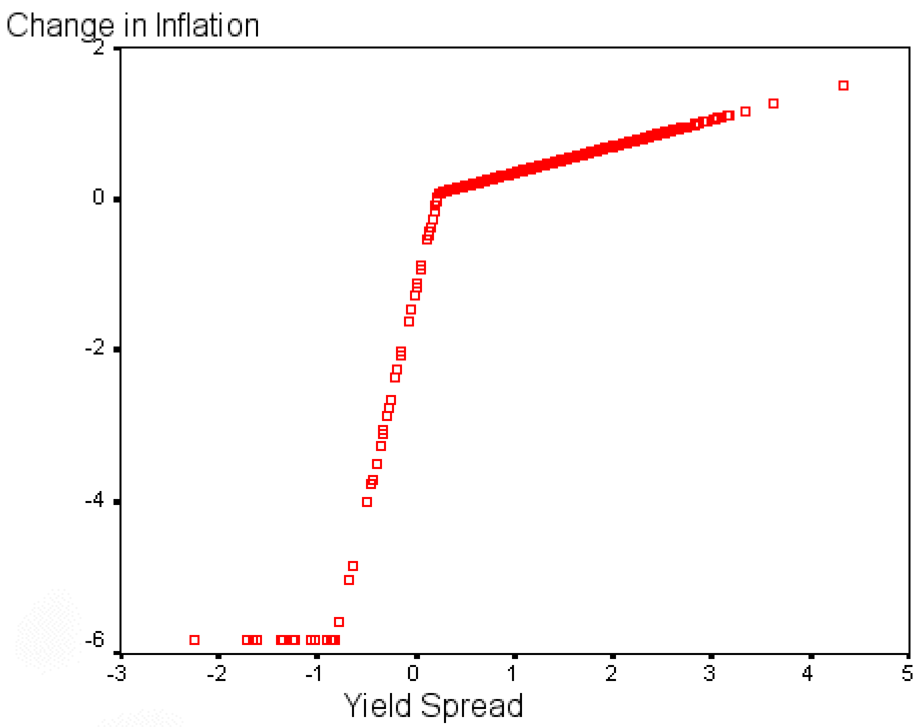

Figure 2: Yield Spread and Inflation Changes: Five Years vs Three Months

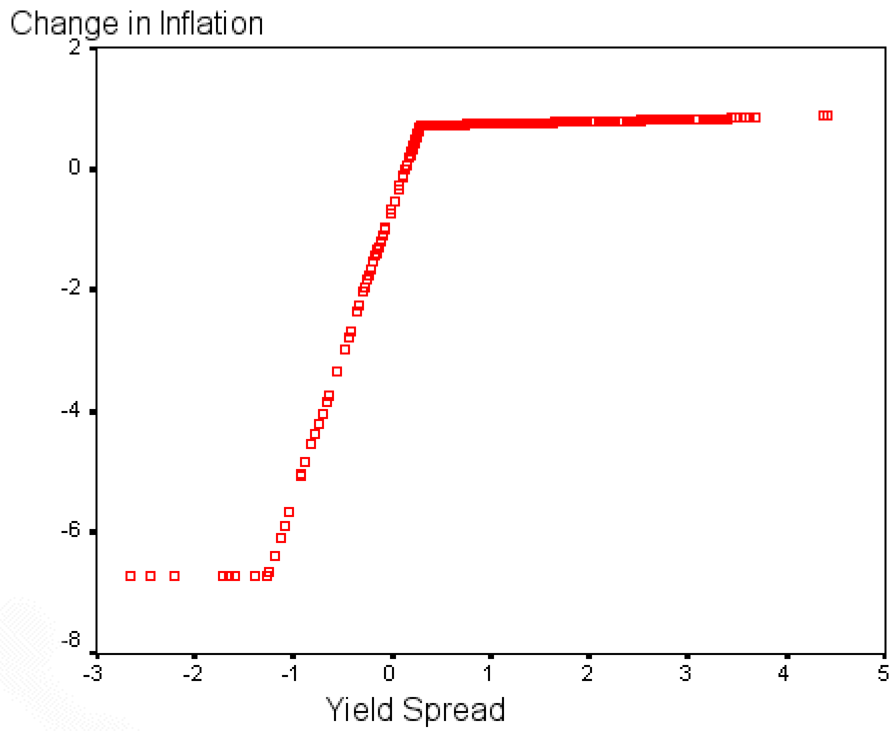

Figure 3: Yield Spread and Inflation Changes: Ten Years vs Three Months